Mathematics Teachers Enrichment Program
MTEP 2012
Sets and Forms

Definitions and Terminology

Sets

A *set* is a collection of distinct (different) objects.

Elements

The individual objects contained in a set are called *elements* or *members*.

Set Notation

When the elements of a set **can** be listed, they are written inside brace brackets, $\{\ \ \}$, separated by commas. The order in which the elements are listed is not important. However, sometimes there may be an advantage to listing the elements in a particular order. It is customary to name a set using an uppercase letter of the alphabet.

Size of a Set

We are generally interested in counting the number of elements in a set. Sometimes we may not be able to list a set but we are still able to count the number of elements in it. For a set A, the number of elements in the set is written $n(A)$.

If $A = \{\text{red, blue, green, yellow}\}$ then $n(A) = 4$ since the set contains four elements.

Examples:

For each given description, list the set and determine the number of elements in the set.

1. A : the days of the week
   Solution:

2. B : the positive odd numbers less than or equal to 20
   Solution:

3. C : the people in this room
   Solution:

4. D : the cards in a standard deck of playing cards
   Solution:

More Terminology

Universal Set

The *Universal Set*, U, is the set containing all elements under consideration in a particular situation. Each of the sets listed in our above examples will be used as universal sets later.

Empty or Null Set

A set containing no elements is called the *empty set* or *null set*. If set P contains no elements we would denote it $P = \varnothing$ or $P = \{\ \}$.

Subsets

If all the elements of set X are also elements of set Y, we say that set X is a *subset* of set Y and write $X \subseteq Y$. We say "X is contained in Y" or "Y contains X". According to this definition, every set is a subset of itself and the empty set is a subset of every set.

Disjoint Sets

When two sets have no elements in common, the sets are said to be *disjoint*.

Equal Sets and Equivalent Sets

Sets that contain exactly the same elements are called *equal* sets. Sets that contain the same number of elements are called *equivalent* sets.

Complement of a Set

The *complement* of a set A is a subset of the universal set U containing all the elements from U that are not in A. We denote the complement of A as A'. It should also be noted that n(U)=n(A)+n(A'). (It could also be noted that complements are disjoint sets.)

Venn Diagrams

Relationships between sets and their subsets can be illustrated using *Venn diagrams*. In such a diagram we use a rectangle to represent the universal set and closed curves, usually circles, to represent the sets in which we are interested.

Examples:

For each of the following, list the sets in set notation. Illustrate the sets on a Venn diagram. The Universal set is listed for you. Classify the subsets as equal, equivalent, disjoint or complements.

5. $U = \{$Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday$\}$
   A : the days of the week starting with the letter S
   B : the days of the week containing 9 letters

6. $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$
   C : the odd integers in U
   D : the even integers in U

7. $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$
   C : the odd integers in U
   E : multiples of 3 in U

Example 8

Using the cards in a standard deck of playing cards, create subsets of U that are (i) disjoint, (ii) complements, (iii) empty, (iv) equal, and (v) equivalent. Write the sets using set notation and illustrate the sets on a Venn diagram.

$$\begin{aligned} U = \{&1\clubsuit,\ 2\clubsuit,\ 3\clubsuit,\ 4\clubsuit,\ 5\clubsuit,\ 6\clubsuit,\ 7\clubsuit,\ 8\clubsuit,\ 9\clubsuit,\ 10\clubsuit,\ J\clubsuit,\ Q\clubsuit,\ K\clubsuit,\\ &1\diamondsuit,\ 2\diamondsuit,\ 3\diamondsuit,\ 4\diamondsuit,\ 5\diamondsuit,\ 6\diamondsuit,\ 7\diamondsuit,\ 8\diamondsuit,\ 9\diamondsuit,\ 10\diamondsuit,\ J\diamondsuit,\ Q\diamondsuit,\ K\diamondsuit,\\ &1\heartsuit,\ 2\heartsuit,\ 3\heartsuit,\ 4\heartsuit,\ 5\heartsuit,\ 6\heartsuit,\ 7\heartsuit,\ 8\heartsuit,\ 9\heartsuit,\ 10\heartsuit,\ J\heartsuit,\ Q\heartsuit,\ K\heartsuit,\\ &1\spadesuit,\ 2\spadesuit,\ 3\spadesuit,\ 4\spadesuit,\ 5\spadesuit,\ 6\spadesuit,\ 7\spadesuit,\ 8\spadesuit,\ 9\spadesuit,\ 10\spadesuit,\ J\spadesuit,\ Q\spadesuit,\ K\spadesuit\} \end{aligned}$$

Solution:

(i) Disjoint subsets

(ii) Complements

(ii) Empty subset

(iv) Equal Sets

(v) Equivalent Sets

And Still More Terminology

For two sets A and B, the **union of** A **and** B is the set, denoted $A \cup B$, containing all the elements of the universal set which are either in set A or in set B or in both sets.

From a previous example,

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

C : the odd integers in U

E : multiples of 3 in U

Illustrate $C \cup E$ on a Venn diagram, list $C \cup E$ in set notation and describe $C \cup E$ in words.

For two sets A and B, the **intersection of** A **and** B is the set, denoted $A \cap B$, containing all the elements of the universal set which are both in set A and in set B.

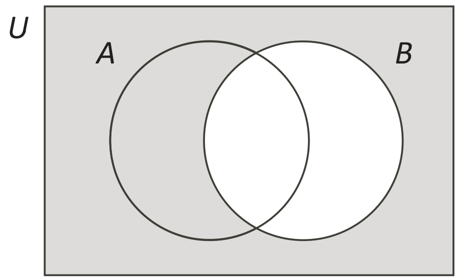
Illustrate $C \cap E$ on a Venn diagram, list $C \cap E$ in set notation and describe $C \cap E$ in words.

Illustrate $(C \cup E)'$ and $(C \cap E)'$on a Venn diagram. List each in set notation.

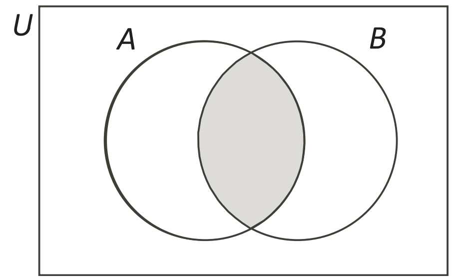
Example 9

The shaded area of each Venn diagram represents some sort of intersection or union or complement or combination of sets A and B. Write an expression that represents the Venn diagram shown.

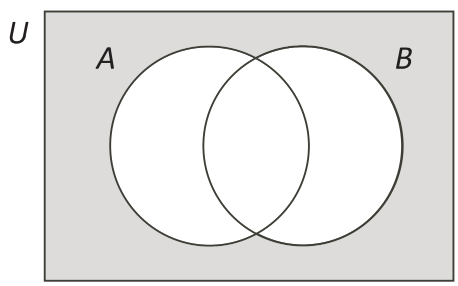
a)



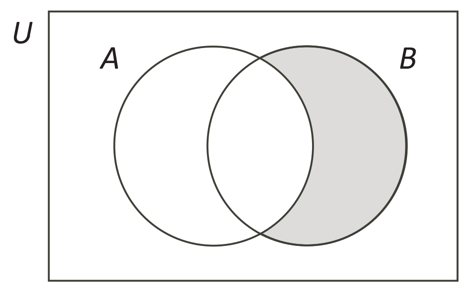
(b)



c)



(d)



Counting Subsets

For disjoint subsets, A and B, $n(A \cup B) = n(A) + n(B)$. This result basically says that there is no intersection between the two subsets and to find the number in the union, we determine the number in each disjoint subset and add the result together.

Example 10

Given the universal set U, and subsets A and B described below, determine $n(A \cap B)$ and $n(A \cup B)$.

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

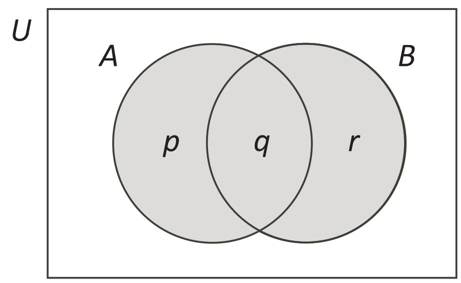
A : the numbers in U with 2 as one of the digits

B : multiples of 7 in U

Counting Subsets (continued)

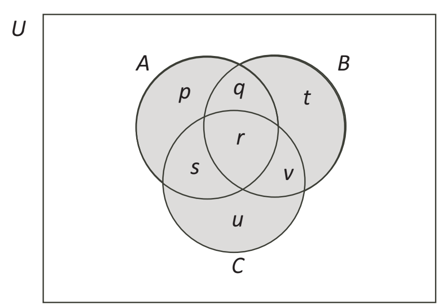
For subsets, A and B, where $A \cap B \neq \varnothing$, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

We will illustrate this result with a Venn diagram.



For subsets, A, B and C,
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$.

We will illustrate this result with a Venn diagram.



Example 11

In a certain village, almost all of the people living there are farmers, 21 of them cultivate cassava and 18 cultivate groundnut. If 9 villagers cultivate both cassava and groundnut, and there are 36 people in the village, how many are not farmers?

Example 12

A school sport's team consists of 100 students. 52 students do field events, 38 students do track events, 41 students swim, 17 students do track and swimming, 22 students do field and track events, 15 students do field events and swimming, and 8 students do all three.

a) Draw a Venn diagram illustrating the data.

b) How many students do only track events?

c) How many students do field events and swimming but not track events?

d) How many students are involved in exactly one of the three events?

e) How many students participate in field events or track events or swimming?