

Mathematics Teachers' Enrichment Program 2011

VARIATIONS

1. X varies directly as Y and inversely as the cube root of Z when $X = 45$, $Y = 27$ and $Z = 729$
 - a) Find the relationship between x , y , and z .
 - b) Calculate x , when $y = 32$, $z = 512$
 - c) Calculate z , when $x = 10$ and $y = 2$
2. V varies directly as the square of R and inversely as H .

When $V = 200$, $H = 25$ and $R = 10$

- a) Find the relationship between V , H and R
 - b) Calculate R , when $V = 40$, $H = 20$
 - c) Calculate H , when $R = 81$ and $V = 405$
3. X is partly constant and partly varies with Y
When $Y = 3$, $X = 7$ and when $Y = 6$, $X = 9$
 - a) Find the relationship between X and Y
 - b) Find X when $Y = 10$
4. D is partly constant and partly varies with V .
When $V = 40$, $D = 150$ and when $V = 54$ and $D = 192$,
 - a) Find the formula connecting D and V
 - b) Hence find D when $V = 73$

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WORD PROBLEMS

Q1) The Gambia insurance company 100 customers. The probability of a customer having an accident in one year is $\frac{1}{1D}$.

The average cost of an accident is D200.

It charges its customer D25 each per year

- a) How many customers will the firm expect to have accidents?
- b) How much will they have to pay out?
- c) How much will it collect fro its customers?
- d) How much profit or loss will they make?

Q2) I sell 'A' papers on Monday. 'B' papers on Tuesday. 'C' papers on Wednesday. The papers are sold for D dalasi cash. If the total selling price is T, write down the formula starting T = and factorize your answer.

Q3) Issa backs a cake, some biscuits and bread. He uses three times as much flour for the biscuits as for the cake. The bread needs one cupful more flour than the cake.

Altogether, he uses eleven cupfuls of flour.

How much flour did he use the bread?

Q4) A cow cost seven times as much as a goat. For D 84.00 I can buy 18 more goats than cows.

How much does a goat cost?

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WORD PROBLEMS

- Q1) The body mass of a man is x -kg. The body masses of his sons are $\frac{5}{6}$ and $\frac{4}{5}$ that of their father.
- (a) Express the sons masses in terms of x
 - (b) If the difference between that masses of the sons is 2.3 kg, find the mass of the father.
- Q2) The sum of two numbers is 22. The sum of $\frac{3}{4}$ of one of the numbers and $\frac{1}{5}$ of the other is 11. Find the two numbers.
- Q3) A man is 5 years older than his wife. Four years ago the ratio of their ages was 7:6. Find their present ages.
- Q4) A certain car covers 10km at a certain speed. If this average speed is reduced by 30km/hr, the car takes the same time to cover a distance of 6km. find the speed of the car in the first part of the journey.

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ELEMENTARY STATISTICS

Q1) The frequency table below is the mark distribution of a class of 35 pupils in an examination.

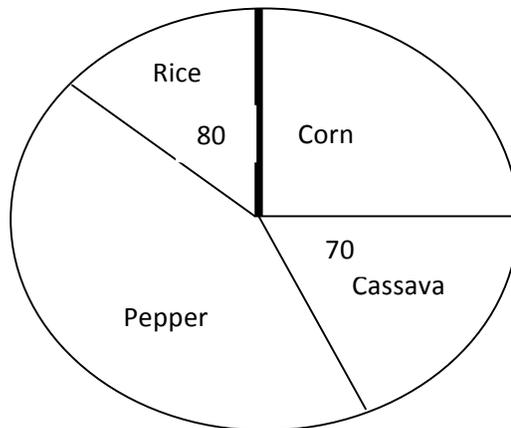
Marks	1	2	3	4	5	6	7	8	9	10
Frequency	2	4	5	6	10	1	2	3	1	1

- a. Find the mean mark for the class
- b. What is the median mark?
- c. What is the modal mark
- d. What is the probability that a pupil chosen from the class obtained:
 - i. A mark less than 5?
 - ii. A mark greater than or equal to 7?

Q2) The following is the result of a survey conducted in a class of a Junior Secondary School to find the favorite soft drink of each pupil in a class.

SOFT DRINK	NO. OF PUPILS PREFERRING SOFT DRINK
Coca cola	6
Pepsi cola	5
Pee cola	8
Fanta	3
Muscatello	5
Mirinda	4
Club cola	6
Sprite	3

- a. Draw a bar chart showing this information, using a scale of 2cm to 1 unit on the vertical axis.
 - b. How many pupils are in the class?
 - c. What percentages of pupils in the class prefer club cola?
- Q3) The mode of five numbers is 1, the median is 5 and the mean is 4. Find the numbers.
- a. What is the range?
 - b. If a number is chosen at random, what is the probability that the number is more than the median?
- Q4) The pie chart shows the distribution of crops on a farm of area 250 hectares.
- a. Find the area of plot planted with corn.
 - b. What fraction of the form is planted with pepper?



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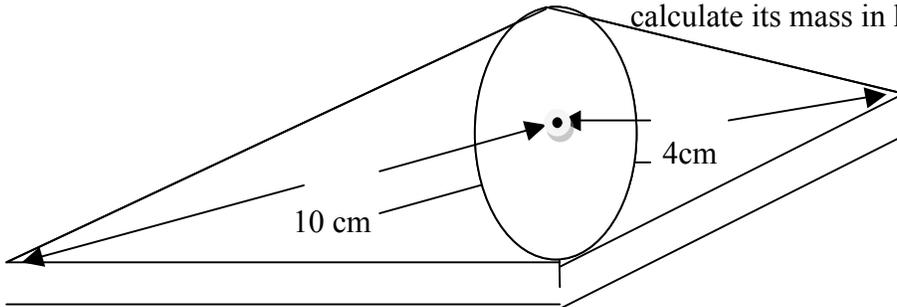
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GEOMETRY

1. The figure below shows a machine part made from two cones of base radius 3cm. The height of the cones are 10cm and 4cm,

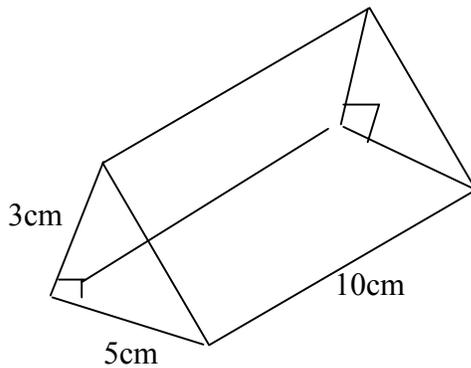
a. Calculate the volume of the machine part in cm^3 .

b. If the machine part is made of brass of density 8g/cm^3 , calculate its mass in kg.



2. A cuboid is divided into two equal right-angle triangle prism.

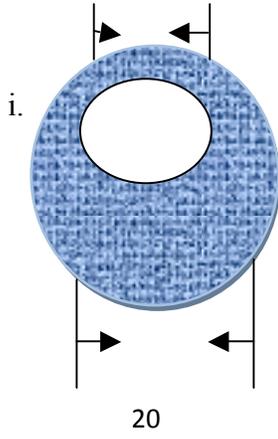
Calculate the volume of the show prism.



3. If the volume of a rectangular-based pyramid is 70cm^3 and the base area is 28cm^2 , calculate the height of the pyramid.

4.

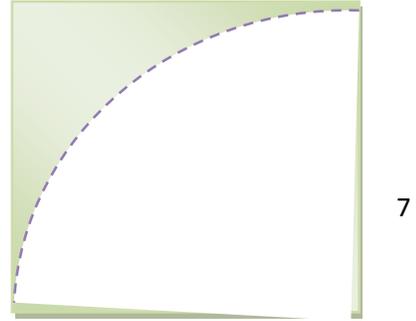
Calculate the shaded area of the following.



10

7

ii.



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TECHNICAL COMMUNICATION

COURSE OUTLINE:

- The course will highlight appropriate communication skills in mathematics classroom.
- Teacher will be taken through technical terms: simply, Evaluate, find the value of..., calculate, Express, solve, locate, Identify, find the least, at most, estimate etc.
- Teachers and instructor will look into some technical questions and solve them accurately.

Some Technical Problems

- 1) From two points on opposite side of a pole 33m high, the angles of elevation of the top of the pole are 53° and 67° . If the two points and the base of the pole are on the same horizontal level, calculate correct to 3 significant figures the distance between the two points.

- 2) (a) A man saved Le 3000 in bank P, whose interest rate was X% per annum and Le 2000 in another bank Q whose interest rate was Y% per annum. His total interest in one year was le 640. If he had saved Le 2000 in bank p and Lle3000 in bank Q for the same period, he would have gained le 20 as additional interest. Find the values of x and y

(b) A man invested D20, 000 in bank A and D25, 000 in bank Bat the beginning of a year. Bank A pays simple interest at a rate of Y% per annum and B pays 1.5y% per annum, if his total interest at the end of the year form the two banks was D6400, find y

- 3) (a) A motorcycle travels a distance of 240km at a uniform speed. If it had been 4km/hr slower, it would have taken 2 hour more to cover the distance. Find its speed.

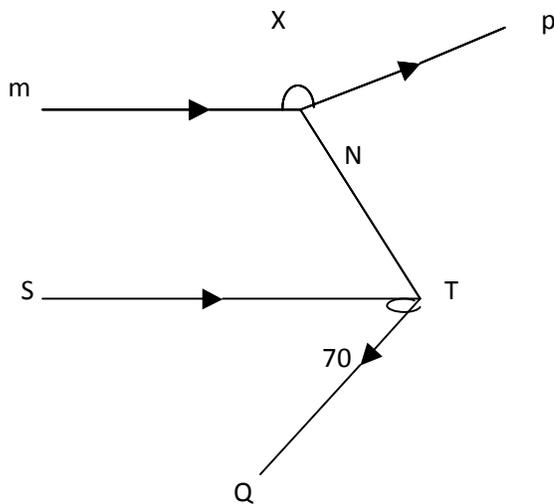
- 3) (b) In Canada, they measure distance in kilometers. One kilometer is about 60% of one mile. Estimate this same speed in:
 - (i) Meters per hour
 - (ii) Meters per second.

- 4) If the equation $px^2 + (p+1)x + p = 0$ has equal roots, identify the value of p for which $p > 0$ (p is greater than zero)

5) A hemispherical tank of diameter which is 10m is filled by water issuing from a pipe of radius 20cm at 2m per second. Calculate, correct to 3 significant figures, the time, in minutes, it takes to fill the tank.

- 6) (a) Two fair dice are tossed together once. Find the probability of getting a total:
- (i) Of at least 9,
 - (ii) Less than 4
 - (iii) At most 4

(b) Find the value of x in the diagram below.



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MANAGEMENT OF MATHEMATICS DEPARTMENT

1. How would you manage the department as a head if:
 - a) There are no materials such as chalk board protector, compasses, set squares for Technical Drawings or Mathematics lessons?
 - b) Textbooks for teachers in the department are insufficient?
 - c) There are insufficient desks/ benches for students?
 - d) The class size is high (say more than 40 students as it is in some schools)?
2. How would you deal with any teacher who sees you not suitable as a head of department or sees you as too harsh?
3. What would be your stand if there is a misunderstanding between teachers in the department?
4. How would you deal with a difficult Principal or Proprietor/ Proprietress of your school?
5. How would you deal with lateness of teachers to classes?

PROBABILITIES

- 1 (a) If a number is chosen at random from the integers 5 to 25 inclusive. Find the probability that the number is a multiple of 5 or 3.
- 1 (b) A bag containing 10 balls that differs only in color; 4 are blue and 6 are red. Two balls are pricked one after the other, with replacement. What is the probability that:
- (i) Both are red
 - (ii) Both are the same?
2. A man has 9 (nine) identical balls in a bag; out of these, 3 are black, 2 are blue and the remaining are red.
- (a) If a ball is drawn at random, what is the probability that it is
- (i) Not blue?
 - (ii) Not red?
- (b) If 2 balls are drawn at random, one after the other, what is the probability that both of them will be
- (i) Black, if there is no replacement?
 - (ii) Blue, if there is a replacement?
3. (a) In a game, a fair die is rolled once and two unbiased coins are tossed once. What is the probability of obtaining 3 and a tail?
- (b) A box contains 10 marble, 7 of which are black and 3 are red, two marbles are drawn one after the pother without replacement. Find the probability of getting:
- (i) A red, then a black
 - (ii) Two black marbles
4. (a) A fair die is tossed once, what is the probability of scoring a:
- (i) 3 or 6
 - (ii) 4 or 5
 - (iii) Neither 6 nor 1

(b) A bag contains 5 red balls, 6 yellow balls and 4 white balls. A ball is picked from the bag at random. Find the probability that the ball is:

- (i) Red or White
- (ii) Yellow or Red
- (iii) Not red
- (iv) Neither red nor yellow

CONSTRUCTIONS

Using a ruler, pencil and a pair of compasses only construct a:

1. (a) Triangle XYZ In which $XY = 55\text{mm}$, $YZ = 82\text{mm}$ $\angle YXZ = 75^\circ$ and $\angle XYZ = 45^\circ$
 - (b) Bisect angle XYZ and XZY.
 - (c) Locate W at the point of the intersection of the bisectors
 - (d) Measure (a.) WZ (b). ZWZ

2. Quadrilateral PQRS in which $PQ = 5\text{cm}$, $QR = 7\text{cm}$, $\angle SPQ = 90^\circ$. $\angle PQR = 120^\circ$, $\angle QRS = 45^\circ$

- 3 (a) Triangle DEF in which $DE = EF = 65\text{mm}$, $DF = 56\text{mm}$
 - (b) What figure have you constructed?
 - (c) Bisect all the angles of the triangle DEF and let the bisectors meet at G
- E. Measure angle EGD

4. Quadrilateral BCDE in which $BC=7\text{cm}$. $CD =5\text{cm}$ and $\angle EBC =90^\circ$, $\angle BCD$ and $\angle CDE=60^\circ$
Measure (1) $\angle BED$ (11) DE

5. Quadrilateral KLMN In which $\angle NKL = 90^\circ$, $\angle KLM =45^\circ$, $KL =5.6\text{cm}$. Measure KN
Which quadrilateral have you constructed?

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STATISTICS

Introduction: The notion of statistics was originally derived from the word “state” since it has been the function of governments to keep records of population, birth, deaths, etc.

The general function of statistics is to develop principles and methods that will help us make decisions in the event of uncertainty.

Using statistics to aid our understanding of problems can, separated into three stages:

1. Identifying the problem to be investigated and investigation of the problem.
2. Collection of data in the form of a list, a table or a graph.
3. Studying the displayed data, drawing conclusions from it, and making decisions for the future.

Q1) The marks scored by 50 students in an examination are as follows:

60	54	40	67	53	73	37	55	62	43
44	69	39	32	45	58	48	67	39	51
46	59	40	52	61	48	23	60	59	47
65	58	74	47	40	59	68	51	50	50
71	51	26	36	38	70	46	40	51	42

- a. Using class intervals 21-30, 31-40, ... prepare a frequency distribution table.
- b. Calculate the mean mark of the distribution.
- c. What percentage of the students scored **more than** 60 %?

Q2) A survey of the number of children in 100 families gave the following distribution.

<i>Number of children in family</i>	<i>0</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>
<i>Number of families</i>	<i>4</i>	<i>36</i>	<i>27</i>	<i>21</i>	<i>5</i>	<i>4</i>	<i>2</i>	<i>1</i>

For this distribution, find:

- a. The mode
- b. The median
- c. The mean

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STATISTICS

Q3) The table shows the amount of money, Rx, spent on books by a group of students.

AMOUNT SPENT	$0 < X \leq 10$	$10 < X \leq 20$	$20 < X \leq 30$	$30 < X \leq 40$	$40 < X \leq 50$	$50 < X \leq 60$
No. OF STUDENTS	0	4	8	12	11	5

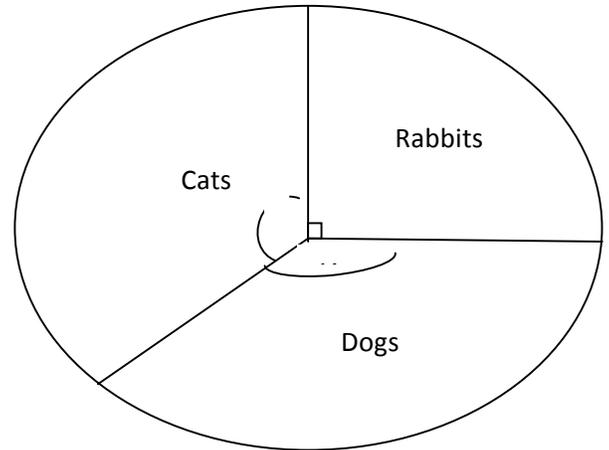
- a. Calculate an estimate of the mean amount of money per student spent on books.
- b. Use the information in the table above to find the values of P, Y and Z in the following cumulative frequency table:

AMOUNT SPENT	$X \leq 10$	$X \leq 20$	$X \leq 30$	$X \leq 40$	$X \leq 50$	$X \leq 60$
CUMULATIVE FREQUENCY	0	4	P	Y	Z	40

- c. Using a scale of 2cm to represent 10 units on each axis, draw a cumulative frequency diagram.
- d. Use your diagram:
 - i. To estimate the median amount spent.
 - ii. To find the upper and lower quartiles, and the inter-quartile range.

Q4) Two hundred children were asked to choose their favorite pet. The results are represented in the pie chart (which is not to scales).

- i. Find the value of x .
- ii. Ho many children chose rabbits?
- iii. What percentage of children chose dogs?



Q5) The table given the frequency distribution of marks obtained by a group of students in a test.

Marks	3	4	5	6	7	8
Frequency	5	$x-1$	x	9	4	1

If the mean is 5, calculate the value of x .

Q6) The table below shows the distribution of marks in a test.

Marks	1-20	21-40	41-60	61-80	81-100
Frequency	8	14	18	48	12

- i. What is the median mark
- ii. What is the class of distribution
- iii. State the modal mark.
- iv. Find the mean of the distribution
- v. Calculate the Standard Distribution

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TRANSFORMATION

Question (1) NOVEMBER 2002

- a. A (3, -6), B (6,-2) and C (x, y) are three points in the xy plane.
If $\frac{1}{3} \vec{OA} + \vec{OB} + \vec{OC} = \vec{AB}$. Find the coordinate of C.
- b. Using a scale of 2cm to 2 units on both axis, draw on a graph sheet, the perpendicular axis OX and OY for the intervals $-10 \leq x < 10$, $-10 \leq y < 10$.
- c. Draw clearly, label the vertices and indicate coordinates as appropriate:-
- i. Triangle PQR with P(4,6), Q (7,9) and $\vec{QR} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$
- ii. The image triangle P₁Q₁R₁ of triangle PQR under a reflection in the line Y=1, where P₁, Q₁ and R₁.
- iv. The image triangle P₂Q₂R₂ of Δ PQR under an anticlockwise rotation of 90 about the origin= where $P \rightarrow P_2$, $Q \rightarrow Q_2$, $R \rightarrow R_2$.
- v. Find R₁R₂.

Question (2) JUNE 2007

- A. Using a scale of 2cm to 4 units on both axis draw on a graph sheet, two-perpendicular- axis OX AND OYT for the intervals $-16 \leq x \leq 16$ and $-16 \leq y \leq 16$.
- B. Draw the Δ ABC with the vertices A (2, 2), B (8, 2) and C (6, 6).
- C. Draw the image Δ A₁B₁C₁ of Δ ABC through a rotation of 180 about the origin, where $A \rightarrow A_1$, $B \rightarrow B_1$, and $C \rightarrow C_1$.
- D. Draw the image Δ A₂B₂C₂ of Δ ABC under the translation by the vector

$$\begin{pmatrix} -8 \\ 8 \end{pmatrix}$$

- E. Draw the image $\triangle A_3B_3C_3$ of $\triangle ABC$ under the reflection in the line $2x-2=0$, where $A_1 \rightarrow A_3$, $B_1 \rightarrow B_3$, and $C_1 \rightarrow C_3$.
- F. Describe the transformation that will map $\triangle ABC$ onto image $\triangle A_3B_3C_3$.
- G. Given that $A(1,2)$, $B(-2,-1)$ and $C(2,3m)$, where m is a constant find
- BA
 - The value of m if $BC = -4 \begin{pmatrix} \\ 5 \end{pmatrix}$

QUESTION (3)

- Using a scale of 2cm to 2 units on both axis, draw on a sheet of graph paper, two perpendicular axis OX and OY for $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.
- Draw on the same graph indicating clearly the coordinates of all vertices;
 - The quadrilateral ABCD with $A(2,2)$, $B(6,2)$, $C(8,8)$ and $D(4,8)$.
 - The image A_1, B_1, C_1, D_1 of quadrilateral ABDC under a reflection in the line $x=0$, where $A \rightarrow A_1$, $B \rightarrow B_1$, $C \rightarrow C_1$, and $D \rightarrow D_1$.
 - The image A_2, B_2, C_2, D_2 of quadrilateral ABDC under a rotation through 180° about the origin, where $A \rightarrow A_2$, $B \rightarrow B_2$, $C \rightarrow C_2$, and $D \rightarrow D_2$.
 - The image A_3, B_3, C_3, D_3 of quadrilateral ABDC under a reflection in the line $y=2$, where $A \rightarrow A_3$, $B \rightarrow B_3$, $C \rightarrow C_3$, and $D \rightarrow D_3$.
- Find the equation of line A_1D_1

Question (4) NOVEMBER 2010

- In triangle DEF, $DE = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $FE = \begin{pmatrix} -6 \\ -4 \end{pmatrix}$. Find DF.

- b. A triangle has vertices A(1,1), B (2,2) and C (5,8).
- i. If the triangle is translated by the vector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ to $A_1B_1C_1$, where where $A \rightarrow A_1$, $B \rightarrow B_1$, $C \rightarrow C_1$, and $D \rightarrow D_1$. Calculate the coordinates of $A_1B_1C_1$.
 - ii. The triangle ABC undergoes a transformation involving a rotation in an anticlockwise direction through 90° about the origin followed by a translation, if the final position is $A_2(-1,0)$, $C_2(-5,3)$, determine the translation vector.
- c. Find the equation of the line which is perpendicular to $y=2x-1$ and passes through the point (2,5).

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CALCULUS

- 1(a) (i) If $y = \sqrt{2 + X^2}$, Find dy/dx
- (ii) $y = e^{4x} + e^{-3x} 2 + X^2$
- (iii) $y = \text{Sin}3x \text{ Cos}2x$
- (b) (i) Given that $y = Ax^3 + B\ln x + C$ where A, B and C are Constants
- Show that $d^3 y/dx^3 = 2/x^2 dy/dx$
- (ii) Given that $y = \text{Sin}3x$. Show that $dy/dx = -3\text{Sin}(3x + \pi/2)$ and deduce that
- $$d^3 y/dx^3 = 3^3 \text{Sin} 3(x + \pi/2)$$
- (iii) Find dy/dx , if $y = x^3 + 3y^4 - y^2 + 2x$
- 2(a) (i) A spherical raindrop is formed by condensation. In an interval of 40 seconds its volume increases at a constant rate from 0.032mm^3 to 0.256mm^3 . Find the rate at which the surface area of a raindrop is increasing when its radius is 0.5mm.
- (ii) A container full of milk takes the form of an inverted right circular cone of height 10mm and base radius 4m. Find in cm s^{-1} , the rate at which the milk level in the container is falling when the height of milk in the container is 5m, given that the milk is flowing from the container at a rate of $100 \text{ cm}^3\text{s}^{-1}$.
- 3(a) (i) If $y = \frac{1 + 2x}{(x + 2)^2}$, Find dy/dx
- (ii) Evaluate $\int_1^5 dx / 2x + 3$, formal integration.
- (iii) By putting $u = 1 + 2x^2$, evaluate $\int_0^1 x / \sqrt{2x + 3} dx$
- 4(a) (i) Using the trapezium rule with 7 ordinates, evaluate, correct to three significant figures, the approximate value of evaluate $\int_0^1 2 / 1 + x^2 dx$

(ii) Using the substitution $u = x + 1$, evaluate $\int_1 x - 1 / (x + 1)^3 dx$

(iii) Evaluate $\int_1 7 + x \sqrt{x^2} / dx$