



# Mathematics Teachers Enrichment Program

## MTEP 2012

### Probability

Rolling a pair of dice is an example of an **experiment**. When each die is rolled there are 6 **equally likely outcomes**, or there is an equal chance that each of the numbers 1,2,3,4,5,6 is going to land face up. The set of *all* possible outcomes of an experiment is called the **sample space** of the experiment. So, for the experiment of rolling a pair of dice, say one white and one black, the sample space consists of 36 possible outcomes. Draw a tree diagram to illustrate the possible outcomes.

List the sample space as a set of ordered pairs:

A **trial** of the experiment is one attempt of the experiment. So for the experiment of rolling a pair of dice, one trial would be rolling the pair of dice once. An **event** is a set of possible outcomes or a **subspace** of the sample space. For example, in the experiment of rolling a pair of dice and the event of having the sum of the roll equal to 3, list the possible outcomes.

Consider two events  $A$  and  $B$ . Each of these events is a subset of the same sample space. The **intersection**,  $A \cap B$ , of two sets  $A$  and  $B$  is the set of all elements common to both  $A$  and  $B$ . The **union**,  $A \cup B$ , is the set of elements that belong to  $A$  or  $B$  or to both  $A$  and  $B$ .

Given a set  $U$  and a subset  $A$  of  $U$ , the **complement** of  $A$ ,  $A'$ , in  $U$  is the set of elements in  $U$  that do not belong to  $A$ . Note that  $n(A) + n(A') = n(U)$ .



If the sample space of an experiment consists of  $N$  equally likely outcomes, the probability of an event  $E$  of the sample space  $S$  is given by:

For example, in the experiment of rolling a single die, each face of the die has an equal chance of landing face up. Determine the probability of rolling a six.

When an event is certain to happen, its probability is 1. When an event is certain *not* to happen, its probability is 0. All probabilities must equal 0 or 1 or lie between them. So, we can write for any probability  $p$ ,  $0 \leq p \leq 1$ .

Given that the probability that an event  $E$  will occur is  $p$ , then the probability that the event  $E$  will *not* occur is  $q = 1 - p$ .

In the experiment of rolling a die, determine the probability of *not* rolling a 6.

Two events,  $E_1$  and  $E_2$ , are said to be **mutually exclusive** if they cannot both occur simultaneously. For example, in the experiment of rolling a pair of dice, two events that are considered mutually exclusive are the event of having the dice sum to 3 and the event of having the dice sum even. Then the probability that two mutually exclusive events  $E_1$  or  $E_2$  will occur is equal to the sum of their probabilities.

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

If the two events  $E_1$  and  $E_2$  are *not* mutually exclusive, then the probability that event  $E_1$  or  $E_2$  will occur is given by the **sum rule**:

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

Two events,  $E_1$  and  $E_2$ , are said to be **independent** when neither event has an influence on the other. For example, in the experiment of rolling a pair of dice, each outcome of a trial is an independent event since the previous roll has no influence on the outcome of the next roll. Then the probability that two independent events will occur is equal to the product of the probabilities of  $E_1$  and  $E_2$ .

$$P(E_1 \cap E_2) = P(E_1) \times P(E_2)$$

In other words, if the first of two tasks can be done in  $a$  ways and, for each of those ways, the second task can be done in  $b$  ways, then by the **product rule**, together the two tasks can be done in  $a \times b$  ways.

In the dice roll example,

- State two events that are mutually exclusive and determine the probability.
- State two events that are not mutually exclusive and determine the probability.
- State two events that are independent and determine the probability.



**Examples:**

1. A card is drawn from a deck of cards. What is the probability that it is:  
a) a 4?                      b) a face card?                      c) a spade?

**Solution:**



2. A number is chosen at random from the integers 5 to 25 inclusive.
- What is the sample space?
  - Find the probability that the number is a multiple of 5 or 7.
  - Find the probability that the number is a multiple of 5 or 3.

**Solution:**



3. In a game, a fair die is rolled once and two unbiased coins are tossed once. What is the probability of obtaining 3 and at least one tail.

**Solution:**

4. A bag contains 5 black marbles and 5 red marbles. A marble is taken from the bag and then a second marble is taken from the bag. Find the probability of drawing 2 red marbles if the first marble is:
- a) replaced before drawing the second                      b) not replaced

**Solution:**

**Exercises:**

- List the sample space for each of the following experiments:
  - A card is drawn from a deck of playing cards and you are interested in the colour of the suit.
  - A card is drawn from a deck of playing cards and you are interested in the suit of the card.
  - A coin is tossed once and a fair die is rolled once.
- In the experiment of rolling a fair die and recording the number, list the possible outcomes for each of the following events:
  - The number is odd.
  - The number is prime.
  - The number is less than 5.
  - The number is more than 7.
- A fair die is rolled once. What is the probability of rolling a:
  - 2 or 5?
  - 3 or 4?
  - Neither 6 or 1?
- A card is drawn from a deck. What is the probability that it is:
  - an ace?
  - a black card?
  - a red face card?
  - a club face card?
  - The  $7\clubsuit$ ?
- A man has 15 marbles in a bag. Six of them are black, 5 are blue and the rest are red.
  - If a marble is drawn at random, what is the probability that it is:
    - Not black
    - Not red
  - If two marbles are drawn at random, one after the other, what is the probability that both of them will be:
    - blue, if there is no replacement
    - red, if there is a replacement

- The following data was collected from a mathematics class.

| Age | Girls | Boys |
|-----|-------|------|
| 13  | 3     | 2    |
| 14  | 5     | 6    |
| 15  | 8     | 9    |
| 16  | 1     | 3    |

If a student is selected from the class at random what is the probability that the student is:

- a boy?
  - 15 years old?
  - a 13 year old girl?
  - a 15 year old boy?
  - less than 15 years old?
  - 17 years old?
- A fair die is rolled three times. What is the probability that the roll is 5 or greater each time?
  - A box contains 10 marbles, 7 of which are black and 3 are red. Two marbles are drawn, one after the other, without replacement. Find the probability of getting:
    - A red, then a black
    - Two black marbles.
  - The letters in the phrase "GO MTEP" are written on separate slips of paper and placed in a bag. What is the probability that two slips drawn simultaneously will both show:
    - vowels?
    - consonants?
  - A multiple choice test has 4 questions. Each question has 5 responses, only one of which is correct. If Robin attempts this test by guessing, what is the probability that she will get all 4 questions right?



11. From the 2006 Examiners' Report  
A man P has 5 red, 3 blue, and 2 white buses. Another man Q has 3 red, 2 blue, and 4 white buses. A bus owned by P is involved in an accident with a bus belonging to Q. Calculate the probability that the two buses are **not** of the same colour.
12. Two dice are thrown, one red and one blue. What is the probability that the number shown on one die is a multiple of the number on the other?  
Note: A multiple of a number is the product of that number multiplied by a whole number. For example, 2 is a multiple of 1 since  $1 \times 2 = 2$ .
13. There are twelve cards numbered 1 to 12. A card is selected at random. What is the probability that:
- the card is either even or a perfect square?
  - the card is even and a perfect square?
14. A bag contains two red marbles and three blue marbles. A second bag contains three red marbles and two blue marbles. A marble is taken from each bag.
- Make a diagram to represent all the possible outcomes.
  - What is the probability that:
    - both marbles are red
    - both marbles are blue?
  - Find the probability that one marble is red and the other is blue.
15. A blue die and a red die are rolled simultaneously. What is the probability of obtaining:
- a total score of 7?
  - a total score of 10?
  - a total score of 7 or 10?
  - a total score not greater than 10?
16. From the 2006 Examiners' Report
- Two pupils are chosen at random from a group of 4 boys and 5 girls. Find the probability that the two pupils chosen would be boys.
  - Twenty percent of the total production of transistors produced by a machine are below standard. If a random sample of 6 transistors produced by the machine is taken, what is the probability of getting:
    - exactly 2 standard transistors?
    - exactly 1 standard transistor?
    - at least 2 standard transistors?
    - at most two standard transistors?
17. A blue die and a red die are rolled simultaneously. A two-digit number is formed with the number on the blue die giving the tens digit and the number on the red die giving the units digit. For example, a 3 on the blue die and 5 on the red die gives the two-digit number 35.
- Draw all the possible outcomes.
  - What is the probability of obtaining a two-digit number which is:
    - greater than 30
    - exactly divisible by 11
    - prime?
  - What is the probability of obtaining a two-digit number that is either a perfect square or exactly divisible by 7?
18. A box contains 5 red marbles and 3 green marbles. Two marbles are drawn one after the other without replacement. Find the probability of drawing first a red, then a green marble.