Mathematics Teachers Enrichment Program
MTEP 2012
Probability Solutions

1. List the sample space for each of the following experiments:

   a) A card is drawn from a deck of playing cards and you are interested in the colour of the suit.

   There are two colour possibilities in a deck of cards. The sample space is: {black,red}.

   b) A card is drawn from a deck of playing cards and you are interested in the suit of the card.

   There are four suits in a deck of cards. The sample space is: {hearts, diamonds, spades, clubs}.

   c) A coin is tossed once and a fair die is rolled once.

   There are six possibilities for the roll of the die and there are two possibilities for the flip of the coin. The sample space consists of 12 items: {H and 1, H and 2, H and 3, H and 4, H and 5, H and 6, T and 1, T and 2, T and 3, T and 4, T and 5, T and 6}.

2. In the experiment of rolling a fair die and recording the number, list the possible outcomes for each of the following events:

   a) The number is odd.
   b) The number is prime.
   c) The number is less than 5.
   d) The number is more than 7.

   a) There are 3 odd numbers on the die. The possible outcomes are: {1,3,5}.

   b) There are three prime numbers on the die. The possible outcomes are: {2,3,5}.

   c) There are 4 numbers on the die less than 5. The possible outcomes are: {1,2,3,4}.

   d) No numbers on the die are more than 7. The possible outcomes are: {}, the empty or null set.

3. A fair die is rolled once. What is the probability of rolling a:

   a) 2 or 5?
   b) 3 or 4?
   c) Neither 6 or 1?

   a) The probability of rolling a 2 is $\frac{1}{6}$ since there is one 2 on the die out of a total of 6 possibilities. The probability of rolling a 5 is the same as scoring a 2. Since these are mutually exclusive events, the probability of rolling a 2 or a 5 is: $P(E) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$.

   b) Since there is only one 3 and one 4 on a die, this question can be done the same way as part (a). The answer is also $\frac{1}{3}$.

   c) To find the probability of rolling neither a 6 or a 1, we can find the probability of scoring a 6 or a 1 and subtract it from 1 since $q = 1 - p$ where p is the probability of the event happening and q is the probability of the event not happening. As in parts (a) and (b), the probability of rolling a 6 or a 1 is $\frac{1}{3}$. Thus, the probability of rolling neither a 6 or 1 is $q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$.

4. A card is drawn from a deck. What is the probability that it is:
   a) an ace? b) a black card? c) a red face card?
   d) a club face card? e) The 7♣ ?

   a) In a standard deck of cards, there are 4 aces and a total of 52 cards. The probability of drawing an ace is $\frac{4}{52} = \frac{1}{13}$.

   b) There are 26 black cards in a deck. The probability of drawing a black card is $\frac{26}{52} = \frac{1}{2}$.

   c) There are 3 face cards per suit and 2 suits that are red. So, there are a total of 6 red face cards. The probability of drawing a red face card is $\frac{6}{52} = \frac{3}{26}$.

   d) Since there are 3 face cards per suit, there are 3 face cards that are clubs. The probability of drawing a club face card is $\frac{3}{52}$.

   e) There is one 7 of clubs in the deck and a total of 52 cards. The probability of drawing the 7 of clubs is $\frac{1}{52}$.

5. A man has 15 marbles in a bag. Six of them are black, 5 are blue and the rest are red.
   a) If a marble is drawn at random, what is the probability that it is:
      i) Not black ii) Not red
   b) If two marbles are drawn at random, one after the other, what is the probability that both of them will be:
      i) blue, if there is no replacement ii) red, if there is a replacement

   a) i) The probability of not drawing a black marble is the same as one minus the probability of drawing a black marble. There are 6 black marbles and 15 marbles in total. The probability of not drawing a black marble is $1 - \frac{6}{15} = \frac{9}{15} = \frac{3}{5}$.

      ii) There are $15 - 6 - 5 = 4$ red marbles. The probability of not drawing a red marble is $1 - \frac{4}{15} = \frac{11}{15}$.

   b) i) First draw: When drawing the first marble, there are 5 blue marbles and a total of 15 marbles. The probability of drawing a blue marble first is $\frac{5}{15} = \frac{1}{3}$.

      Second draw: Since there is no replacement, there are now a total of 14 marbles, 4 of which are blue. The probability of drawing the second blue marble is $\frac{4}{14} = \frac{2}{7}$.

      Then, using the product rule, the probability of drawing two blue marbles is $\frac{1}{3} \times \frac{2}{7} = \frac{2}{21}$.

      ii) First draw: When drawing the first marble, there are 4 red marbles and a total of 15 marbles. The probability of drawing a red marble is $\frac{4}{15}$.

      Second draw: Since the first red marble was replaced, when the second marble is drawn there are still 4 red marbles and a total of 15 marbles. So the probability of drawing the second red marble is the same as the first.

      Then, using the product rule, the probability of drawing two red marbles is $\frac{4}{15} \times \frac{4}{15} = \frac{16}{225}$.

6. The following data was collected from a mathematics class.

Age	Girls	Boys
13	3	2
14	5	6
15	8	9
16	1	3

If a student is selected from the class at random what is the probability that the student is:

a) a boy? b) 15 years old? c) a 13 year old girl?
d) a 15 year old boy? e) less than 15 years old? f) 17 years old?

a) There are a total of $2+6+9+3=20$ boys in the class and $20+3+5+8+1=37$ students in the class. The probability of selecting a boy is $\frac{20}{37}$.

b) There are $8+9=17$ students that are 15 years old. The probability of selecting a 15 year old student is $\frac{17}{37}$.

c) There are 3 girls in the class that are 13 years old. The probability of selecting a 13 year old girl is $\frac{3}{37}$.

d) There are 9 boys in the class that are 15 years old. The probability of selecting a 15 year old boy is $\frac{9}{37}$.

e) There are $3+2+5+6=16$ students in the class that are less than 15 years old. The probability of selecting a student that is less than 15 years old is $\frac{16}{37}$.

f) There are 0 students in the class that are 17 years old. The probability of selecting a 17 year old student is $\frac{0}{37}=0$.

7. A fair die is rolled three times. What is the probability that the roll is 5 or greater each time?

Each time the die is rolled there are only two numbers that are greater or equal to 5, (5 or 6), So the probability of rolling a number greater than or equal to 5 is $\frac{2}{6}=\frac{1}{3}$. Each time the die is rolled is an independent event. So the probability of rolling the die three times and it showing a 5 or greater each time is: $p=\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}=\frac{1}{27}$.

8. A box contains 10 marbles, 7 of which are black and 3 are red. Two marbles are drawn, one after the other, without replacement. Find the probability of getting:

a) A red, then a black b) Two black marbles.

a) First draw: When drawing the first marble, there are 3 red marbles and a total of 10 marbles. The probability of drawing a red marble is $\frac{3}{10}$.

Second draw: Since there is no replacement, there are now a total of 9 marbles, 7 of which are black. The probability of drawing a black marble after a red marble is $\frac{7}{9}$.

Then, the probability of drawing a red marble then a black marble is $\frac{3}{10}\times\frac{7}{9}=\frac{21}{90}=\frac{7}{30}$.

b) First draw: When drawing the first marble, there are 7 black marbles and a total of 10 marbles. The probability of drawing the first black marble is $\frac{7}{10}$.

Second draw: After the first black marble is drawn, there are only 6 black marbles left. Since there is no replacement, there are only 9 marbles left in the box. The probability of drawing a second black marble is $\frac{6}{9}=\frac{2}{3}$.

Then, the probability of drawing two black marbles, one after the other, is $\frac{7}{10}\times\frac{2}{3}=\frac{14}{30}=\frac{7}{15}$.

9. The letters in the phrase "GO MTEP" are written on separate slips of paper and placed in a bag. What is the probability that two slips drawn simultaneously will both show:
   a) vowels? b) consonants?

   a) We can find the probability of drawing two slips simultaneously by pretending that the slips are drawn one after the other without replacement.

   First draw: In the phrase "GO MTEP", there are 2 vowels and a total of 6 letters. The probability of drawing the first vowel is $\frac{2}{6} = \frac{1}{3}$.

   Second Draw: After the first vowel has been drawn, there is only one vowel left and 5 slips in total. The probability of drawing the second vowel is $\frac{1}{5}$.

   Then the probability of drawing two vowels simultaneously is $\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$.

   b) First Draw: There are a total of 4 consonants in the phrase and 6 letters in total. The probability of drawing the first consonant is $\frac{4}{6} = \frac{2}{3}$.

   Second Draw: After the first consonant has been chosen, there will only be 3 consonants left and 5 slips of paper. The probability of drawing the second consonant is $\frac{3}{5}$.

   Then, the probability of drawing two consonants simultaneously is $\frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$.

10. A multiple choice test has 4 questions. Each question has 5 responses, only one of which is correct. If Robin attempts this test by guessing, what is the probability that she will get all 4 questions right?

    For each question, since there is only one correct answer and 5 possible responses, the probability of getting the question correct is $\frac{1}{5}$. Then, since each question is independent of the others, the probability that Robin will guess each of the 4 question correctly is $\frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = \frac{1}{625}$.

11. From the 2006 Examiners' Report

    A man P has 5 red, 3 blue, and 2 white buses. Another man Q has 3 red, 2 blue, and 4 white buses. A bus owned by P is involved in an accident with a bus belonging to Q. Calculate the probability that the two buses are **not** of the same colour.

    To find the probability that the buses are not of the same colour, we first find the probability that the buses are of the same colour.

    Both red: Mr. P has 5 red buses and a total of 10 buses, Mr. Q has 3 red buses and a total of 9 buses. The probability that the buses involved in the accident are both red is $\frac{5}{10} \times \frac{3}{9} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

    Both blue: Mr. P has 3 blue buses and a total of 10 buses, Mr. Q has 2 blue buses and a total of 9 buses. The probability that the buses involved in the accident are both blue is $\frac{3}{10} \times \frac{2}{9} = \frac{6}{90} = \frac{1}{15}$.

    Both white: Mr. P has 2 white buses and a total of 10 buses, Mr. Q has 4 white buses and a total of 9 buses. The probability that the buses involved in the accident are both white is $\frac{2}{10} \times \frac{4}{9} = \frac{8}{90} = \frac{4}{45}$.

    So the probability of both buses involved in the accident are both red or both blue or both white is $\frac{1}{6} + \frac{1}{15} + \frac{4}{45} = \frac{29}{90}$. And the probability that the two buses are not of the same colour is $1 - \frac{29}{90} = \frac{61}{90}$.

12. Two dice are thrown, one red and one blue. What is the probability that the number shown on one die is a multiple of the number on the other?

    Note: A multiple of a number is the product of that number multiplied by a whole number. For example, 2 is a multiple of 1 since $1 \times 2 = 2$.

    There are a total of 36 possible outcomes. From the sample space there are 22 outcomes with one die as a multiple of the other, {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,4), (2,6), (3,1), (3,3), (3,6), (4,1), (4,2), (4,4), (5,1), (5,5), (6,1), (6,2), (6,3), (6,6)}.

    The probability of that the number shown on one die is a multiple of the number on the other is $\frac{22}{36} = \frac{11}{18}$.

13. There are twelve cards numbered 1 to 12. A card is selected at random. What is the probability that:

    a) the card is either even or a perfect square?

    b) the card is even and a perfect square?

    a) The sample space consists of 12 items, 6 of which are even. The probability of selecting an even number is $\frac{6}{12} = \frac{1}{2}$.

    From the numbers 1 to 12, there are 3 numbers that are perfect squares, namely 1, 4 and 9. So the probability of selecting a card that is a perfect square is $\frac{3}{12} = \frac{1}{4}$.

    Since 4 is also an even number, these events are not mutually exclusive so we have to subtract the probability of selecting 4 from our answer to avoid double counting the probability of selecting 4. Thus, the probability of selecting a number that is even or a perfect square is $\frac{1}{2} + \frac{1}{4} - \frac{1}{12} = \frac{8}{12} = \frac{2}{3}$.

    b) There is only one number that is both even and a perfect square, namely 4. The probability of selecting 4 is $\frac{1}{12}$. $\therefore$ the probability of selecting an even number and a perfect square is $\frac{1}{12}$.

14. A bag contains two red marbles and three blue marbles. A second bag contains three red marbles and two blue marbles. A marble is taken from each bag.

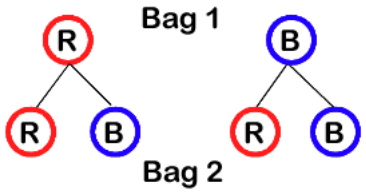
    a) Make a diagram to represent all the possible outcomes.

    b) What is the probability that:

    i) both marbles are red ii) both marbles are blue?

    c) Find the probability that one marble is red and the other is blue.

    a)



    b) i) From the first bag, there are 2 red marbles and a total of 5 marbles. From the second bag, there are 3 red marbles and a total of 5 marbles. The probability of selecting a red marble from each bag is $\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$.

    ii) From the first bag, there are 3 blue marbles and in the second bag there are 2 blue marbles. Both bags have a total of 5 marbles. The probability of selecting two blue marbles, one from each bag, is $\frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$.

    c) The probability of a different colour from each bag is 1 minus the probabilty that the two colours selected are the same. Using the answers from part (b), the probability is $1 - \frac{6}{25} - \frac{6}{25} = \frac{13}{25}$.

    Alternatively, we could determine the probability of a red from bag 1 followed by a blue from bag 2 and add it to the probability of a blue from bag one and a red from bag 2. This probability is $\frac{2}{5} \times \frac{2}{5} + \frac{3}{5} \times \frac{3}{5} = \frac{4}{25} + \frac{9}{25} = \frac{13}{25}$, as before.

15. A blue die and a red die are rolled simultaneously. What is the probability of obtaining:

a) a total score of 7? b) a total score of 10?
c) a total score of 7 or 10? d) a total score not greater than 10?

a) There are a total of 36 possible outcomes when rolling a pair of dice. There are 6 possibilities where the dice sum to 7, {(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)}. The probability of obtaining a total score of 7 is $\frac{6}{36} = \frac{1}{6}$.

b) There are 3 ways to roll the dice and obtain a total score of 10, {(4,6), (5,5), (6,4)}. The probability of obtaining a total score of 10 is $\frac{3}{36} = \frac{1}{12}$.

c) The events of obtaining a total score of 7 or 10 are mutually exclusive. The probability of obtaining a score of 7 or 10, using the answers from (a) and (b), is $\frac{1}{6} + \frac{1}{12} = \frac{1}{4}$.

d) We can determine the probability of obtaining a total score not greater than 10 by finding the probability of obtaining a total score greater than 10 and subtracting it from 1. There are 3 ways to obtain a total score greater than 10, {(5,6), (6,5), (6,6)}. The probability of obtaining a total score greater than 10 is $\frac{3}{36} = \frac{1}{12}$. The probability of obtaining a total score not greater than 10 is $1 - \frac{1}{12} = \frac{11}{12}$.

16. From the 2006 Examiners' Report

a) Two pupils are chosen at random from a group of 4 boys and 5 girls. Find the probability that the two pupils chosen would be boys.

b) Twenty percent of the total production of transistors produced by a machine are below standard. If a random sample of 6 transistors produced by the machine is taken, what is the probability of getting:

i) exactly 2 standard transistors? ii) exactly 1 standard transistor?
iii) at least 2 standard transistors? iv) at most 2 standard transistors?

a) First pick: There are 4 boys and a total of 9 students. The probability of choosing a boy first is $\frac{4}{9}$.

Second pick: After one boy is chosen, there are only 3 boys left and 8 students left to choose from. The probability of choosing the second boy is $\frac{3}{8}$.

The probability of choosing two boys is $\frac{4}{9} \times \frac{3}{8} = \frac{12}{72} = \frac{1}{6}$.

b) The probability of choosing a standard transistor is 80% or $\frac{4}{5}$ and the probability of choosing a below standard transistor is 20% or $\frac{1}{5}$.

i) To find the probability of choosing exactly two standard transistors we first have to find the number of possible ways to select 2 transistors from the 6 to be standard, then the other 4 will be below standard. We can do this by using the "choose" function: $\binom{6}{2} = \frac{6!}{2!(6-2)!} = 15$. So, there are 15 ways to have 2 standard transistors and 4 below standard when choosing 6 transistors. Using S for standard and B for below standard the 15 possibilities can be listed to verify the calculation:
$BBSSSS$, $BSBSSS$, $BSSBSS$, $BSSSBS$, $BSSSSB$,
$SBBSSS$, $SBSBSS$, $SBSSBS$, $SBSSSB$, $SSBBSS$, $SSBSBS$, $SSBSSB$,
$SSSBBS$, $SSSBSB$, $SSSBBS$.

Since the choosing of the transistors are independent events, the probability of selecting exactly two standard transistors is

$$15 \times \frac{4}{5} \times \frac{4}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = 15 \times \frac{4}{5} \times \frac{4}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = 15 \times \frac{16}{15625} = \frac{48}{3125}.$$

ii) To find the probability of choosing exactly one standard transistor, we need to determine the number of possible ways to select the standard transistor from the 6 transistors. There are 6 ways to choose which transistor will be standard (since $\binom{6}{1} = 6$). Then in order to have exactly one standard transistor, we will also need to have 5 below standard transistors. Then the probability of choosing exactly one standard transistor is $6 \times \frac{4}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = 6 \times \frac{4}{15625} = \frac{24}{15625}$.

iii) The probability of having at least two standard transistors is the same as one minus the probability of have exactly one or no standard transistors. We already found the probability of having exactly one standard transistor in part (b). The probability of having no standard transistors is the same as the probability of having 6 below standard transistors. The probability of having 6 below standard transistors is $\frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = \frac{1}{15625}$.

So, the probability of having at least 2 standard transistors is $1 - \frac{1}{15625} - \frac{24}{15625} = \frac{624}{625}$.

iv) The probability of having at most two standard transistors is the same as the probability of having no standard transistors or exactly one standard transistor or exactly 2 standard transistors. The events are mutually exclusive so we can add the individual probabilities. The probability of having at most 2 standard transistors is $\frac{1}{15625} + \frac{24}{15625} + \frac{48}{3125} = \frac{53}{3125}$.

17. A blue die and a red die are rolled simultaneously. A two-digit number is formed with the number on the blue die giving the tens digit and the number on the red die giving the units digit. For example, a 3 on the blue die and 5 on the red die gives the two-digit number 35.

a) Draw all the possible outcomes.

b) What is the probability of obtaining a two-digit number which is:

i) greater than 30 ii) exactly divisible by 11 iii) prime?

c) What is the probability of obtaining a two-digit number that is either a perfect square or exactly divisible by 7?

a) The sample space is {11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66}.

b) i) From the sample space, there are 24 numbers that are greater than 30 and a total of 36 numbers. The probability of forming a number greater than 30 is $\frac{24}{36} = \frac{2}{3}$.

ii) There are 6 numbers from the sample space that are divisible by 11, {11, 22, 33, 44, 55, 66}. The probability of forming a number that is divisible by 11 is $\frac{6}{36} = \frac{1}{6}$.

iii) There are 8 numbers in the sample space that are prime, {11, 13, 23, 31, 41, 43, 53, 61}. The probability of forming a prime number is $\frac{8}{36} = \frac{2}{9}$.

c) There are 4 numbers in the sample space that are perfect squares, {16, 25, 36, 64} and 6 numbers that are divisible by 7, {14, 21, 35, 42, 56, 63}. These events are mutually exclusive, so the probability of forming a number that is a perfect square or divisible by 7 is $\frac{4}{36} + \frac{6}{36} = \frac{10}{36} = \frac{5}{18}$.

18. A box contains 5 red marbles and 3 green marbles. Two marbles are drawn one after the other without replacement. Find the probability of drawing first a red, then a green marble.

First draw: There are 5 red marbles and a total of 8 marbles. The probability of drawing a red marble first is $\frac{5}{8}$.

Second draw: Since the first marble was not replaced, there are now only 7 marbles, 3 of which are green. The probability of drawing a green marble after a red marble is $\frac{3}{7}$.

Then the probability of drawing a red marble followed by a green marble is $\frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$.