



# Mathematics Teachers Enrichment Program

## MTEP 2013

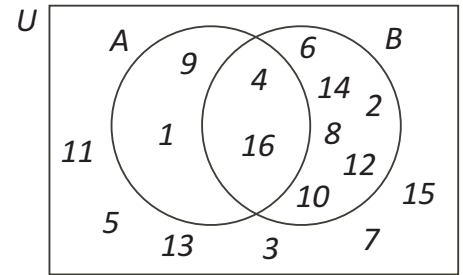
### Sets and Forms: Exercises

- For each of the following sets, determine the number of elements in the set.
  - $A$ : The positive integers less than 100 that are divisible by 3.
  - $B$ : Two digit numbers whose digits add to 9.
  - $C$ : The possible outcomes of flipping a coin.
- For each of the following subsets, state the complement and determine the number of elements in the complement.
  - $U$ : The integers less than 100.  
 $A$ : The positive integers less than 100 that are divisible by 3.
  - $U$ : The letters of the alphabet.  
 $B$ : The consonants in the alphabet.
  - $U$ : The set of positive two digit numbers.  
 $C$ : Positive two digit numbers whose digits add to 9.
  - $U$ : The days of the week.  
 $D$ : The days of the week ending with the letters "day".
- Given  $U$ : The integers from 1 to 100.
  - State two disjoint subsets.
  - State two subsets that are complements.
  - State two equivalent subsets.
  - State two equal subsets.
- Using terms *equal*, *disjoint*, and *subset*, describe the relationship between each of the following sets. Define a suitable universal set and draw a Venn diagram for each situation.
  - $R$ : All rectangles.  
 $C$ : All circles.
  - $E$ : All figures with sides of equal length.  
 $T$ : Equilateral triangles.
  - $N$ : Countries in North America.  
 $A$ : Countries in Africa.
  - $E$ : Even integers.  
 $D$ : Numbers that are divisible by two.



5.  $U$ ,  $A$  and  $B$  are three sets shown in the Venn diagram.

- Describe set  $U$  in set notation and in words.
- Describe set  $A$  in set notation and in words.
- Describe set  $B$  in set notation and in words.
- List the members of the set  $A \cup B$ ,  $A \cap B$ ,  $(A \cup B)'$ ,  $A'$  and  $B'$ .
- Describe set  $A \cap B$  in words.
- Describe set  $(A \cup B)'$  in words.
- Using the numbers from the earlier parts, verify 
$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$



6. Given  $U$ : the set of positive integers 1 to 15

$A$ : The even integers in from 1 to 15.

$B$ : The multiples of 3 from 1 to 15.

- Using set notation, list the elements in  $A$ ,  $B$ ,  $A'$ ,  $B'$ ,  $A \cap B$  and  $A \cup B$ .
- Draw a Venn diagram to illustrate the relationship between  $A$ ,  $B$  and  $U$ .
- Determine the number of elements in each of the following:
  - $A \cup B$
  - $(A \cup B)'$
  - $A \cap B$
  - $A \cap B'$

7. Given  $U$ : The letters in the alphabet.

$A$ : The letters in the word “BANJUL”.

$B$ : The different letters in the word “WATERLOO”.

- Using set notation, list the elements in  $A$ ,  $B$ ,  $A \cap B$  and  $A \cup B$ .
- Determine  $n(A \cup B)$  using  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ .
- Draw a Venn diagram to illustrate the relationship between  $A$ ,  $B$  and  $U$ .
- Determine the number of elements in each of the following:
  - $A \cup B$
  - $A'$
  - $A \cap B$
  - $(A \cap B)'$
  - \*Challenging\*  $A' \cap B'$
  - $(A \cup B)'$

8. Given  $U$ : The whole numbers from 1 to 25.

$A$ : The multiples of 2.

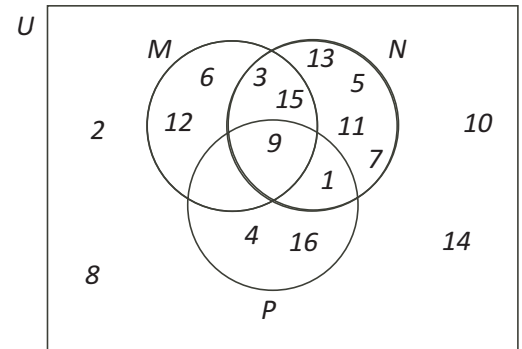
$B$ : The multiples of 3.

$C$ : The multiples of 5.

- Determine  $n(A)$ ,  $n(B)$ ,  $n(C)$ ,  $n(A \cap B)$ ,  $n(A \cap C)$ ,  $n(B \cap C)$ ,  $n(A \cap B \cap C)$ .
- Determine  $n(A \cup B \cup C)$  using 
$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C).$$
- Determine the number of numbers in  $U$  that are not divisible by 2, 3 and 5.
- Determine the number of numbers in  $U$  that are multiples of 2 and 3 but not 5.
- Draw a Venn diagram and use it to confirm your results from above.



9.  $M$ ,  $N$  and  $P$  are three sets shown in the Venn diagram with universal set  $U$ .



a) Describe set  $U$ ,  $M$ ,  $N$  and  $P$  in set notation and in words.

b) Determine  $n(U)$ ,  $n(M)$ ,  $n(N)$ , and  $n(P)$ .

c) Determine  $n(M \cap N)$ ,  $n(M \cap P)$ ,  $n(N \cap P)$ , and  $n(M \cap N \cap P)$ .

d) Determine  $n(M \cup N \cup P)$  using

$$n(M \cup N \cup P) = n(M) + n(N) + n(P) - n(M \cap N) - n(M \cap P) - n(N \cap P) + n(M \cap N \cap P).$$

e) Verify your result from (d) using the Venn diagram.

f) Determine the subset  $N' \cap (M \cup P)$  and  $N' \cap (M \cap P)$ .

10. There are 40 students in a class, 13 of these students like math and 26 of them like science. If 5 of the students in the class do not like math or science, how many students liked both math and science.

11. The Swiss embassy in Ottawa, Canada has 65 employees. Of these workers, 47 speak German, 35 speak Italian, and 20 speak both German and Italian. How many embassy employees speak neither German nor Italian. Illustrate the situation with a Venn diagram.

12. Out of 60 teachers attending MTEP 2012, 45 are married, 25 are women. If 23 of the men are married, how many women are unmarried.

13. In a recent survey, students were asked to state whether or not they had heard of two popular musical artists: Justin Bieber and Katie Perry. Of the students surveyed, 72% said they had heard of Justin Bieber, 53% said they had heard of Katie Perry, and 41% had heard of both.

a) What percentage of the students surveyed had heard of Justin Bieber but not Katie Perry?

b) What percentage of the students surveyed had heard of Katie Perry but not Justin Bieber?

c) What percentage of the students surveyed had not heard of either?

14. At a market, 58 people were asked whether they liked fruits or vegetables. 8 people said they liked neither, 38 people said they liked fruit, and 30 people liked vegetables. How many of those asked said they liked both fruit and vegetables?



15. In a class of 32 students, 22 do mathematics and 18 do chemistry. If 5 students do neither, how many students do mathematics but not chemistry?
16. Of 700 students who have travelled out of The Gambia, 400 have travelled to Senegal and 250 have travelled to the Ivory Coast. If 150 have travelled to both countries, how many of the students have not travelled either country?
17. The 29 students in Mr. G's math class use a variety of forms of transportation to get to school. Twenty of them sometimes arrive at school in a car, 12 bicycle to school some of the time, and 16 take the bus occasionally. If four students use all three options, six either bike or take the bus, ten come by car or bike, and nine arrive by car when they do not come by bus, how many always use some other method of getting to school?
18. By examining 75 records at D. D. Collegiate, the following data was collected:
  - 12 students studied Mathematics, History and Geography
  - 27 studied Mathematics and Geography
  - 20 studied Mathematics and History
  - 21 studied History and Geography
  - 45 studied Mathematics
  - 36 studied History
  - 48 studied Geography
  - a) From the records examined, how many students studied at least one of Mathematics, History or Geography? Illustrate your answer with a Venn diagram?
  - b) How many students studied Mathematics and Geography but not History?
  - c) How many students took exactly one of the subjects?
  - d) How many students did not take any of Mathematics, History or Geography?